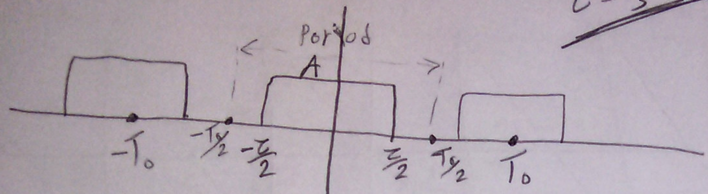
1

Sec $\boxed{3}$ نظم اتصالات

* Find and sketch the magnitude spectrum of the following functions using Complex F.S and Calculate the avg. power

$\tau = \frac{1}{3} T_0$



$$C_n = \frac{1}{T_0} \int^{T_0} g_p(t)\, e^{-jn\omega_0 t}$$

$$|C_n| = \sqrt{a_n^2 + b_n^2}$$

$$g_p(t) = \begin{cases} A & -\frac{\tau}{2} \le t \le \frac{\tau}{2} \\ 0 & \text{otherwise} \end{cases}$$

$$C_n = \frac{1}{T_0} \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} A\, e^{-jn\omega_0 t}\, dt$$

$$= \frac{A}{T_0} \cdot \frac{1}{-jn\omega_0} \left[e^{-jn\omega_0 t} \right]_{-\frac{\tau}{2}}^{\frac{\tau}{2}}$$

$$= \frac{A}{T_0} \cdot \frac{1}{-jn\omega_0} \left[e^{-jn\omega_0 \frac{\tau}{2}} - e^{jn\omega_0 \frac{\tau}{2}} \right]$$

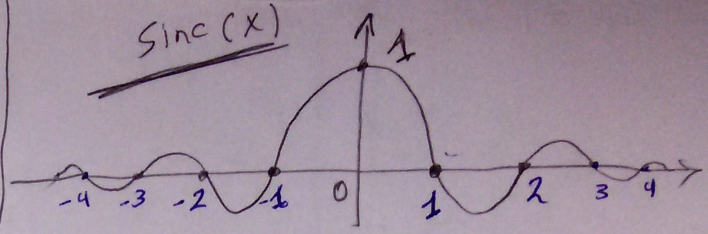
Note $\boxed{\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}}$

$$= \frac{2A}{2\pi n} \left[\frac{e^{jn\omega_0 \frac{\tau}{2}} - e^{-jn\omega_0 \frac{\tau}{2}}}{2j} \right]$$

$$\boxed{C_n = \frac{A}{n\pi} \sin\left(n\omega_0 \frac{\tau}{2}\right)}$$

Note $\boxed{\operatorname{sinc}[x] = \frac{\sin(\pi x)}{\pi x}}$

sinc (x)



sinc (x) = 0 when x is integer and $x \neq 0$

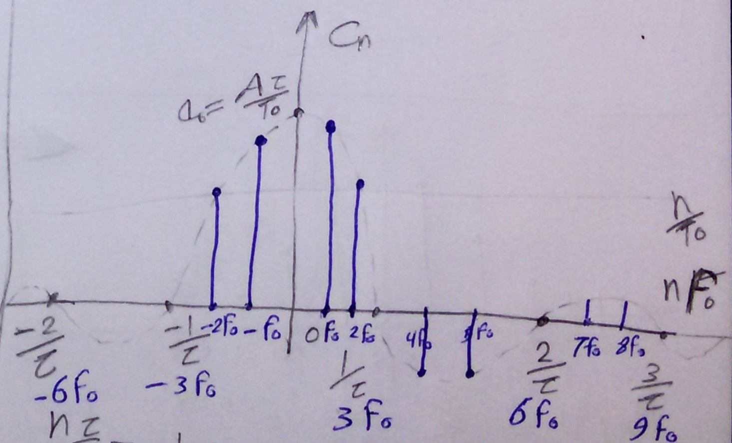
$$C_n = \frac{A}{n\pi} \sin\left(n \frac{2\pi}{T_0} \frac{\tau}{2}\right)$$

$$C_n = \frac{A\tau}{T_0} * \frac{\sin\left(\frac{n\pi\tau}{T_0}\right)}{\frac{n\pi\tau}{T_0}}$$

$$\boxed{C_n = \frac{A\tau}{T_0} \operatorname{sinc}\left(\frac{n\tau}{T_0}\right)}$$

$$\therefore g_p(t) = \sum_{n=\infty}^{\infty} C_n \cdot e^{+jn\omega_0 t}$$

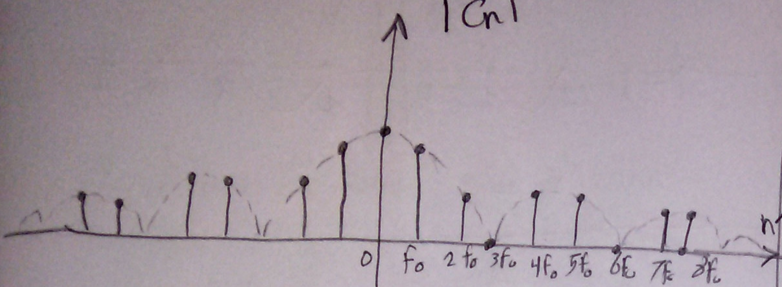
$$= \frac{A\tau}{T_0} \sum_{n=\infty}^{\infty} \operatorname{sinc}\left(\frac{n\tau}{T_0}\right) e^{+jn\omega_0 t}$$



$$\frac{n\tau}{T_0} = 1, 2, 3, \ldots \quad \operatorname{sinc}\left(\frac{n\tau}{T_0}\right) = 0$$

$$\frac{n}{T_0} = \frac{1}{\tau}, \frac{2}{\tau}$$

$C_n = |C_n| \cdot e^{j\theta_n}$

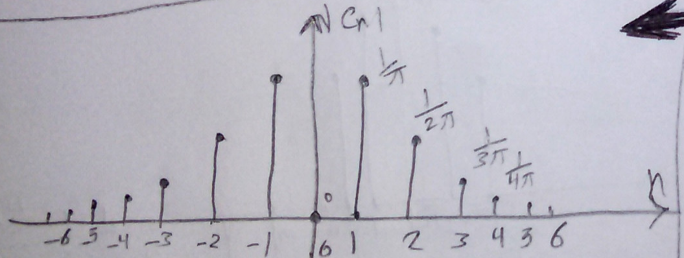


$$*P_{avg} = \frac{1}{T_0}\int^{T_0} g^2(t)\,dt$$

$$= \frac{1}{T_0}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} A^2\,dt$$

$$= \frac{A^2}{T_0}\cdot t \Big|_{-\frac{\tau}{2}}^{\frac{\tau}{2}}$$

$$\boxed{P_{avg} = \frac{A^2 \tau}{T_0} \quad \text{watts}}$$



$$g_p(t) = \sum_{n=-\infty}^{\infty} \frac{j}{n\pi}\, e^{+jn\omega_0 t}$$

$$P_{avg} = \frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{t^2}{\pi^2}\,dt$$

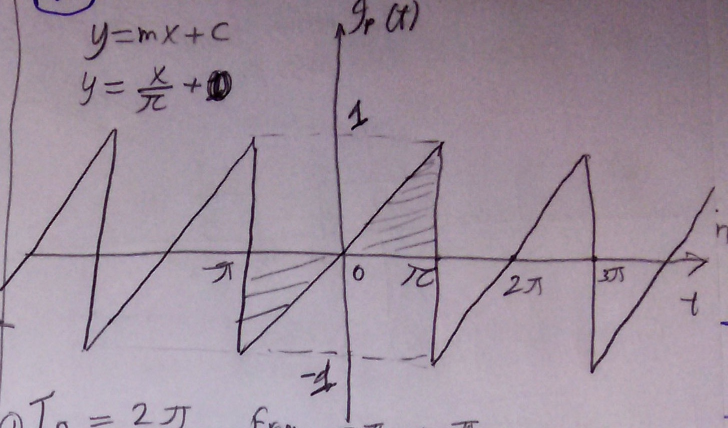
$$= \frac{1}{3*2\pi^3}\,|t^3|_{-\pi}^{\pi}$$

$$= \frac{1}{3} \quad \text{watt}$$

(2)

$y = mx + c$

$y = \frac{x}{\pi} + 0$



① $T_0 = 2\pi$ from $-\pi \to \pi$

② $g_p(t) = \frac{t}{\pi}$ $\quad -\pi \leq t \leq \pi$

③ $g_p(t)$ is odd $\quad a_n = 0,\ a_0 = 0$

$$b_n = \frac{1}{T_0}\int^{T_0} g_p(t)\cdot \sin(n\omega_0 t)\,dt$$

$$\boxed{\omega_0 = \frac{2\pi}{T_0} = 1}$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{t}{\pi}\sin(n\omega_0 t)\,dt$$

$$= \frac{1}{2\pi^2}\left[\frac{-t\cos(n\omega_0 t)}{n\omega_0} + \int\frac{\cos(n\omega_0 t)}{n\omega_0}dt\right]_{-\pi}^{\pi}$$

$u = t$

$du = 1$

$dv = \sin(n\omega_0 t)$

$v = -\frac{\cos(n\omega_0 t)}{n\omega_0}$

$$= \frac{1}{2\pi^2}\left[\left[\frac{-\pi\cos(n\omega_0\pi)}{n\omega_0} - \frac{\pi\cos(n\omega_0 - \pi)}{n\omega_0}\right] + \frac{\sin(n\omega_0 t)}{n^2\omega_0^2}\Big|_{-\pi}^{\pi}\right]$$

$\sin(n\pi) = 0$

$\cos(n\pi) = (-1)^n$

$$= \frac{1}{2\pi^2}\left[\frac{-2\pi\cos(n\pi)}{n\omega_0} + 0\right]$$

$$\boxed{b_n = \frac{(-1)^{n+1}}{n\pi}}$$

$C_n = |C_n|\, e^{j\theta_n}$

$\theta_n = \tan^{-1}\frac{b_n}{a_n}$

$\theta_n = \frac{\pi}{2}$

$|C_n| = |b_n| = \frac{1}{n\pi}$

$|C_1| = \frac{1}{\pi}$

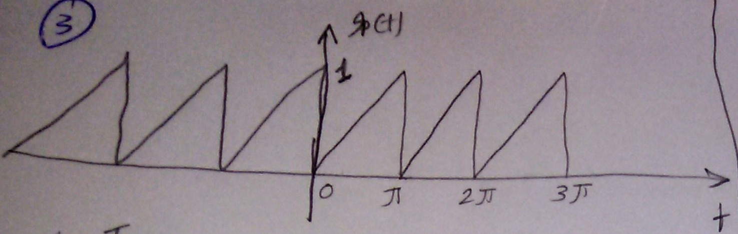
$|C_2| = \frac{1}{2\pi}$

$|C_3| = \frac{1}{3\pi}$

$C_n = j\,\frac{1}{n\pi}$

2 Sec $\boxed{2}$ نظم اتصالات

(3)



* $T_0 = \pi$ from $0 \to \pi$
* $g_p(t) = \frac{t}{\pi}$
* not odd nor even

$$\boxed{w_0 = \frac{2\pi}{T_0} = 2}$$

$$C_n = \frac{1}{T_0}\int^{T_0} g_p(t)\, e^{-jnw_0t}\, dt$$

$$= \frac{1}{\pi^2}\int_0^{\pi} t\, e^{-jnw_0t}\, dt$$

$$u = t \qquad dv = e^{-jnw_0t}$$

$$du = 1 \qquad v = \frac{1}{-jnw_0} e^{-jnw_0t}$$

$$C_n = \frac{1}{\pi^2}\left[\frac{-t}{jnw_0} e^{-jnw_0t} + \int_0^{\pi} \frac{e^{-jnw_0t}}{jnw_0}\, dt\right]_0^{\pi}$$

$$C_n = \frac{1}{\pi^2}\left[\left(\frac{-\pi}{jnw_0} e^{-j2n\pi}\right) + \frac{1}{j2n}\left[\frac{e^{-j2nt}}{-j2n}\right]_0^{\pi}\right]$$

$$C_n = \frac{1}{\pi^2}\left[\left(\frac{-\pi}{j2n} e^{-j2n\pi}\right) - \frac{1}{4n^2}\underbrace{\left(e^{-j2n\pi} - 1\right)}_{\to 0}\right]$$

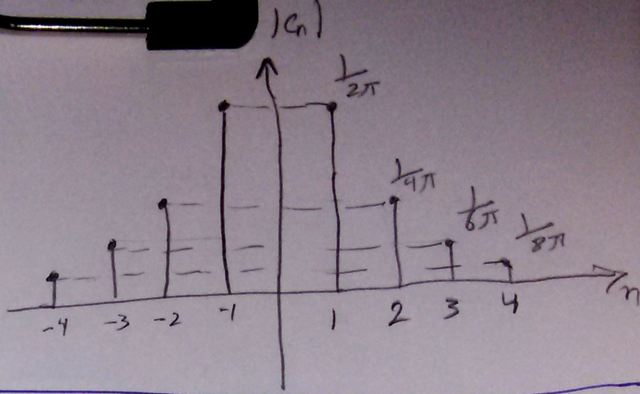
$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$e^{-jn\pi} = \underbrace{\cos(2\pi n)}_{\to 1} - j\underbrace{\sin(2n\pi)}_{\to 0}$$

$$C_n = \frac{1}{\pi^2}\left[\frac{-\pi}{j2n}\right]$$

$$C_n = \frac{-1}{j2n\pi} = \frac{j}{2n\pi}$$

$$|C_n| = \frac{1}{2n\pi}$$



وصورت حل رقم $\boxed{2}$ بحافوق C_n

$$P_{avg} = \frac{1}{\pi}\int_0^{\pi} \frac{t^2}{\pi^2}\, dt$$

$$= \frac{1}{3\pi^3}\left[t^3\right]_0^{\pi}$$

$$= \frac{1}{3}$$